The Mathematics of CoPilot

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1 Units

CoPilot performs all calculations, and stores all data in the database, using a standard set of units. Data is converted to the units preferred by the user whenever data is read from, or written to, the user interface. The standard units are:

Distance nautical miles

Altitude feet

Airspeed knots

Climb Rate feet per minute

Fuel US gallons

Fuel Flow US gallons per hour

Pressure inches of mercury

Temperature Celsius degrees

2 Density Altitude

All user specified altitudes (a) are first converted to density altitude before use. Pressure altitude (a_p) is first determined (from the pressure) with the formula:

$$a_p = a + 145442.2 * \left(1 - \left(\frac{pressure}{29.92126}\right)^{0.190261}\right) \tag{1}$$

Density altitude (y) is then determined (from the pressure altitude and the temperature) using the formula:

$$y = a_p + \frac{288.15 - 0.0019812 * a_p}{0.0019812} * \left(1 - \left(\frac{288.15 - 0.0019812 * a_p}{temp + 273.15}\right)^{0.234969}\right) \tag{2}$$

3 Aircraft Data

3.1 Aircraft Cruise

The user enters a table of cruise information that consists of altitude (a), true airspeed (t), and fuel flow (f) data sets. The data is assumed to be for standard conditions of pressure and temperature, therefore the specified altitude is equal to the density altitude. The data is stored in the aircraft database after conversion from the user units to the standard units.

A mathematical model of the cruise performance is determined by performing a least squares curve fit to produce a quadratic equation that describes the true airspeed (knots) and fuel flow (US gallons per hour) as a function of density altitude (feet). The quadratic coefficients for true airspeed (t_a, t_b, t_c), and fuel flow (f_a, f_b, f_c) are stored in the aircraft database.

The actual true airspeed (t) and fuel flow (f) at a particular density altitude (y) can then be calculated as follows:

$$t = t_a * y^2 + t_b * y + t_c \tag{3}$$

$$f = f_a * y^2 + f_b * y + f_c \tag{4}$$

3.2 Aircraft Climb

The user enters a table of climb information that consists of altitude (a), climb rate (r), indicated airspeed (i), and fuel flow (f) data sets. The data is assumed to be for standard conditions of pressure and temperature, therefore the specified altitude is equal to the density altitude. The data is stored in the aircraft database after conversion from the user units to the standard units.

A mathematical model of the climb performance is determined by performing a least squares curve fit to produce a quadratic equation that describes the climb rate (feet per hour), true horizontal airspeed (feet per hour), and fuel flow (US gallons per hour) as a function of density altitude (feet). In order to calculate the true horizontal airspeed, the indicated airspeed is converted to true airspeed. Mach is first determined (from the indicated airspeed and the pressure altitude) using the formula:

$$m = \sqrt{5 * \left(\frac{29.92126 * \left(\left(1 + 0.2 * \left(\frac{i}{661.4786}\right)^2\right)^{3.5} - 1 \right)}{29.92126 * (1 - 6.8755856 * 10^{-6} * a_p)^{5.2558797}} + 1 \right)^{\frac{2}{7} - 1}} \tag{5}$$

True airspeed (t) is then determined (from mach and temperature) using the formula:

$$t = m * 38.967854 * \sqrt{temp + 273.15} \tag{6}$$

The horizontal component of the true airspeed (in feet per hour) is then determined using the formula:

$$h = \sqrt{(t * 6076.11549)^2 - r^2} \tag{7}$$

The climb rate (r) is converted to feet per hour before the quadratic coefficients are calculated.

The quadratic coefficients for climb rate (r_a, r_b, r_c), true horizontal airspeed (h_a, h_b, h_c), and fuel flow (f_a, f_b, f_c) are stored in the aircraft database. The time (hours) to climb from density altitude y_1 to density altitude y_2 is:

$$time = \int_{y_1}^{y_2} \frac{1}{r_a * y^2 + r_b * y + r_c} dy \tag{8}$$

The distance (feet) required to climb from density altitude y_1 to density altitude y_2 is:

$$distance = \int_{y_1}^{y_2} \frac{h_a * y^2 + h_b * y + h_c}{r_a * y^2 + r_b * y + r_c} dy \tag{9}$$

The fuel (US Gallons) required to climb from density altitude y_1 to density altitude y_2 is:

$$fuel = \int_{y_1}^{y_2} \frac{f_a * y^2 + f_b * y + f_c}{r_a * y^2 + r_b * y + r_c} dy \tag{10}$$

CoPilot evaluates these integrals by performing a numerical integration using Simpson's method (with 8 segments).